7. W. Nowacki, "Couple-stresses in the theory of thermoelasticity," Bull. Acad. Pol. Sci., Ser. Sci. Tech., 14, No. 2, 97-106 (1966).
8. W. Nowacki, "Couple-stresses in the theory of thermoelasticity," Bull. Acad. Pol. Sci., Ser. Sci. Tech., 14, No. 3, 203-212 (1966).
9. W. Nowacki, "Couple-stresses in the theory of thermoelasticity," Bull. Acad. Pol. Sci., Ser. Sci. Tech., 14, No. 8, 505-513 (1966).
10. Vl. N. Smirnov, "Equations of generalized thermoelasticity of a Cosserat medium," Inzh.-Fiz. Zh., 39, No. 4, 716-723 (1980).
11. A. I. Lur'e, Theory of Elasticity [in Russian], Nauka, Moscow (1970).
12. J. Ignaczak, "Direct determination of stresses from the stress equation of motion in elasticity," Arch. Mech. Stoss., 11, No. 5, 671-678 (1959).
13. J. Ignaczak, "A completeness problem for stress equation of motion in linear elasticity theory," Arch. Mech. Stoss., 15, No. 2, 225-234 (1963).
14. S. De Groot and P. Mazur, Non-Equilibrium Thermodynamics, American Elsevier (1962).

EXCITATION OF A TEMPERATURE WAVE BY A RECTANGULAR THERMAL SURFACE PULSE

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We calculate the temperature field in a metal under the action of a rectangular thermal surface impulse with a fixed total energy and varying duration. It is shown that for a given duration of this impulse, conditions are created which ensure a maximal shift of the melting isotherm toward the interior of the metal.

In the present work we consider a thermophysical process in a metal in regimes when the following condition holds for the thermal pulse applied at the boundary:

$$W = Ft = \text{const}, \tag{1}$$

where F is the thermal flux density, which is constant during the time of its action t. The condition W = constant can be realized in various ways: from a short pulse of a high-density thermal flux, to an extended pulse for a low density of the thermal flux. Condition (1) essentially describes a multitude of pulses which differ by parameters F and t but have the same parameter W.

An analysis shows that the action of thermal pulses which differ in parameters F and t but have the same parameter W has appreciably different results on the metal.

For a long pulse duration the high thermal conductivity characteristic for metals ensures the transfer of the heat flux far into the metal. Therefore, the long pulse excites a deep but weak heating of the metal whose temperature field is extended over a large region. Towards the end of the pulse, the melting isotherm remains near the surface of the metal because of the weak heating. For short pulses of the same energy W, on the other hand, the metal is heated to large temperatures, and the temperature field is concentrated near the surface of the metal. In this case, the melting isotherm towards the end of the pulse also remains near the surface of the metal but for a different reason, because of the spatial concentration of the temperature field.

Clearly, in the intermediate conditions between long and short duration at a given energy W, the melting isotherm will be displaced by the largest amount. The aim of the present work is to substantiate this assertion quantitatively because of its importance in the analysis of the appropriate scientific and applied problems.

In the solution of the problem formulated above we shall limit ourselves to the analysis of a one-dimensional thermophysical process, and neglect the phase transformations. The process will be approximated by the problem of excitation of a temperature field (or a temperature wave) by a rectangular thermal pulse:

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TABLE 1. Values of M ($m^4 \cdot sec/J^2$) and N (m^3/J) for Some Metals

Metal	M	N	Metal	M	N
Al	$1{,}55 \cdot 10^{-15}$	$2{,}66 \cdot 10^{-10}$	Ag	$1{,}88 \cdot 10^{-16}$	$1{,}18 \cdot 10^{-10}$
Fe	$4{,}22 \cdot 10^{-16}$	$5{,}44 \cdot 10^{-11}$	Cd	$5{,}86 \cdot 10^{-15}$	$3{,}98 \cdot 10^{-10}$
Ni	$2{,}48 \cdot 10^{-16}$	$4{,}90 \cdot 10^{-11}$	Sn	$5{,}20 \cdot 10^{-15}$	$3{,}32 \cdot 10^{-10}$
Cu	$1{,}00 \cdot 10^{-16}$	$8{,}20 \cdot 10^{-11}$	W	$2{,}51 \cdot 10^{-17}$	$3{,}00 \cdot 10^{-11}$
Zn	$1{,}78 \cdot 10^{-15}$	$2{,}10 \cdot 10^{-10}$	Pb	$2{,}22 \cdot 10^{-14}$	$5{,}75 \cdot 10^{-10}$
Mo	$9{,}25 \cdot 10^{-17}$	$4{,}60 \cdot 10^{-11}$	Co	$4{,}96 \cdot 10^{-16}$	$4{,}95 \cdot 10^{-11}$

TABLE 2. Values of Functions η, β_1, and β_2

η	β_1	β_2	η	β_1	β_2
0	0	5,73	0,91	0,405	1,91
0,1	0,00082	5,37	0,92	0,438	1,86
0,2	0,0038	5,00	0,93	0,466	1,76
0,3	0,0102	4,61	0,94	0,491	1,70
0,4	0,0240	4,25	0,95	0,528	1,63
0,5	0,0455	3,83	0,96	0,562	1,53
0,6	0,0780	3,41	0,97	0,614	1,48
0,7	0,133	2,98	0,98	0,681	1,37
0,8	0,224	2,51	0,99	0,753	1,26
0,9	0,380	1,98	1	1	1

$$\frac{\partial T(x,\ t)}{\partial \tau} = a\,\frac{\partial^2 T(x,\ t)}{\partial x^2};\quad -\lambda\,\frac{\partial T(0,\ \tau)}{\partial x} = F;\quad \frac{\partial T(\infty,\ \tau)}{\partial x} = 0;$$
$$0 \leqslant x \leqslant \infty;\quad T(x,\ 0) = T_0;\quad 0 \leqslant \tau \leqslant t, \tag{2}$$

where t is a parameter of the pulse, i.e., its total duration, and τ is the instantaneous time of its action.

The justification of the applicability of this approximation will be given in the concluding part of this work.

The solution of problem (2) is known to be [1]

$$\text{ierfc}\left(\frac{x}{2\sqrt{a\tau}}\right) = \frac{C_V[T(x) - T_0]}{2F}\sqrt{\frac{a}{\tau}}. \tag{3}$$

The important point in this solution for $\tau = t$ is the point $x = x_m$ at which the temperature corresponds to the melting of the metal: $T(x_m) = T_m$. One can also choose another characteristic point $x = x_m^*$ at which the temperature coincides with the reduced melting temperature:

$$T(x_m^*) = T_m^* = T_m + \frac{L_V}{C_V}. \tag{4}$$

At the point $x = x_m^*$, noting that $\tau = t$ and $F = W/t$, Eq. (3) can be written in the form

$$\text{ierfc}\left(\frac{x_m^*}{2\sqrt{at}}\right) = \frac{C_V(T_m^* - T_0)\sqrt{at}}{2W}. \tag{5}$$

A similar expression can also be written for the point x_m.

Denoting the argument of the special function

$$x_m^*/2\sqrt{at} = \alpha \tag{6}$$

and solving Eq. (5) with respect to the time of duration of the thermal pulse, we find

$$t = 4(\text{ierfc}\,\alpha)^2\,\frac{W^2}{aC_V^2(T_m^* - T_0)^2}. \tag{7}$$

Noting, moreover, that according to (6), $\sqrt{at} = x_m^*/2\alpha$, Eq. (5) can also be solved for the coordinate of the characteristic point of the thermal field:

$$x_m^* = 4\alpha \operatorname{ierfc} \alpha \frac{W}{C_V (T_m - T_0)} \cdot \tag{8}$$

Equations (7) and (8) are solvable since the special function ierfc α is tabulated [1] and the functions

$$4 (\operatorname{ierfc} \alpha)^2 = \mu, \quad 4\alpha \operatorname{ierfc} \alpha = \gamma \tag{9}$$

can be assumed known. The α dependence of these functions is shown in Fig. 1. It is seen that for some $\alpha = \alpha_0$, the functions reach their maximum values $\gamma = \gamma_0$, $\mu = \mu_0$. The values α_0, γ_0, and μ_0 can, with sufficient accuracy, be obtained from the tables of the function ierfc α [1] or from Fig. 1:

$$\alpha_0 = 0.430; \quad \mu_0 = 0.222; \quad \gamma_0 = 0.405. \tag{10}$$

Using these data, Eqs. (7) and (8) can be written in their final form, which is useful for practical calculations:

$$x_m^* = \eta N W; \quad t = \beta M W^2, \tag{11}$$

where

$$N = \frac{\gamma_0}{C_V (T_m^* - T_0)}; \quad M = \frac{\mu_0}{a C_V^2 (T_m^* - T_0)^2}; \tag{12}$$

$$\eta = \frac{\gamma}{\gamma_0}; \quad \beta = \frac{\mu}{\mu_0}. \tag{13}$$

The values of N and M for some metals at $T_0 = 300°K$ are given in Table 1. It is seen that these quantities differ appreciably for different metals.

It should be emphasized that the functions η and β in (11) depend on α, and form a two-valued correspondence; one value of η corresponds to two values of β. For the convenience of practical calculations, Table 2 gives the corresponding values of these functions. Each line contains one value of η and the two corresponding values of β_1 and β_2.

It is seen from Table 2 that for $\eta = 1$, $\beta_1 = \beta_2 = 1$. For a pulse with a fixed parameter W, these parameters determine the conditions when the pulse excites the greatest depth of heating (melting) of the metal. It follows from Eq. (11) that this optimum regime is defined by the following formulas:

$$x_m^* = NW, \quad t_0 = MW^2. \tag{14}$$

In what follows it is expedient to divide Eq. (11) into two regions of the parameter t ($t < t_0$ and $t > t_0$) in view of the two-valued correspondence between the functions η and β:

$$x_m^* = \eta N W, \quad t = \beta_1 M W^2, \quad t < t_0, \tag{15}$$

$$x_m^* = \eta N W, \quad t = \beta_2 M W^2, \quad t > t_0. \tag{16}$$

An important law follows from the data in Table 2 and from the equations above: For $\eta \to 0$ $\beta_1 \to 0$, and $\beta_2 \to 5.73$. This indicates that in the region $t < t_0$, as the duration of the impulse increases, one has $x_m^* \to 0$ asymptotically, and in the region $t > t_0$ when the duration increases up to

$$t_m = 5.73 M W^2, \tag{17}$$

one has $x_m^* = 0$.

Equation (17) defines the maximum duration of the thermal pulse with a given parameter W, at which the isotherm $T = T_m^*$ (or $T = T_m$) reaches the boundary of the metal. For longer durations of the thermal pulse ($t > t_m$), the melting point is not reached at the boundary, and the pulse has no erosive results.

By analyzing Eq. (17) and taking into account the data in Table 1 it is seen that for a given W, the difference in the values of t_m for different metals can be substantial, and can reach three orders of magnitude. This

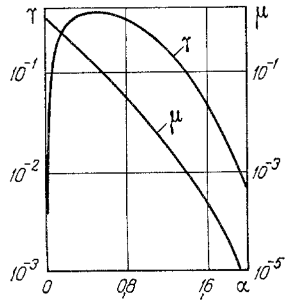

Fig. 1

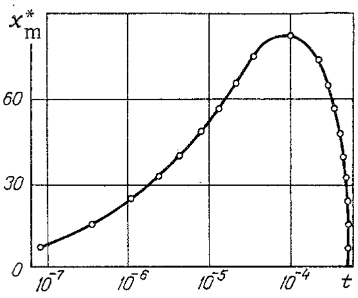

Fig. 2

Fig. 1. Dependence on α of the functions γ and μ.

Fig. 2. Dependence of the displacement of the melting isotherm x_m^* (μm) on the duration of the applied rectangular thermal pulse t (sec) for copper and $W = 1\ J/mm^2$.

expresses the individual properties of the metals with respect to the action of a thermal pulse with a given parameter W.

Figure 2 gives the results of calculation of x_m^* and t from Eqs. (14)-(17) and from the data of Table 2. The calculations were carried out for copper and $W = 1\ J/mm^2$.

The dependence $x_m^* = x_m^*(t)$ shown in Fig. 2 illustrates in a clear and specific form the fact that, for a pulse with a given energy W, there is indeed a duration t_0 for which the melting isotherm is displaced towards the interior of the metal by the greatest amount.

In conclusion one should discuss the possibility of application of the results. First of all, the starting problem (2) does not take into account the melting process, and this results in the approximate form of the results. However, a corresponding analysis of the more general Stephan problem, taking into account the melting process, which was considered in [2] showed that the increase in accuracy is of the order of 10%, and this only pertains to the vicinity of the maximum of the curve $x_m^* = x_m^*(t)$ where the discrepancy is greatest. Consequently, the obtained dependence $x_m^* = x_m^*(t)$ can, with acceptable accuracy, be used to find the shift of the melting front. This conclusion is justifiable only for rectangular pulses in the region $t < t_m$ when t_m is not too large, and the process remains one-dimensional.

In addition, the starting problem (2) does not take into account evaporation. A short pulse in which, for a given W, the heat flux density becomes large excites a high-temperature field and causes intensive evaporation. The solution of the problem which takes into account this process has been solved by us elsewhere [3, 4], and leads in many ways to different conclusions. The increased accuracy of the shift of the melting isotherm (or melting front), however, consists of less than 10%. This result also indicates the applicability of the obtained results for qualitative discussions.

It should also be noted that the obtained dependence $x_m^* = x_m^*(t)$ is of interest in applications. Figure 2 indicates that for short rectangular pulses with a given parameter W, the position of the melting isotherm becomes more shallow and consequently, the pulses cause less damage. It is also seen that to obtain the maximum depth of the melting isotherm for the same W, it is necessary only to ensure an appropriate duration of the pulse. One is also interested in the durations $t > t_m$ when the rectangular pulses with a given parameter W cannot bring about any phase transformation or metal damage.

NOTATION

C_V, volume specific heat; a, thermal diffusivity; L_V, volume specific heat of melting; T_m, melting temperature of the metal; T_0, initial temperature; and W, surface energy density of the heat pulse.

LITERATURE CITED

1. A. V. Lykov, Theory of Thermal Conduction [in Russian], Vysshaya Shkola, Moscow (1967), pp. 575-577.
2. J. Ready, Effects of High-Power Laser Radiation, Academic Press (1971).
3. A. G. Goloveiko, Inzh.-Fiz. Zh., 13, 215 (1967).
4. A. G. Goloveiko, Inzh.-Fiz. Zh., 17, 272 (1969).